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Equating coefficients of like powers, $2a_2=-1$, $3a_3=a_2$, $4a_4=-a_3$, and $5a_5=a_4$.

Solving for a_2, a_3, a_4, a_5, and substituting in (1) and (2),

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-$$

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INTEGRATION OF ELLIPTIC INTEGRALS.

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[Continued from March Number.]

Writing $s+s^{-1}$ for $\cos2\varphi$ in (B), (C), (D), (E) we get

$$2(1+e^2-es-es^{-1})^{-\frac{1}{2}}=A_0+A_1(s+s^{-1})+A_2(s^2+s^{-2})+A_3(s^3+s^{-3})$$
$$+\ldots\ldots+A_n(s^n+s^{-n})+\ \ldots\ldots\ldots(37).$$

$$2(1+e^2-es-es^{-1})^{-\frac{3}{2}}=B_0+B_1(s+s^{-1})+B_2(s^2+s^{-2})+B_3(s^3+s^{-3})$$
$$+\ldots\ldots+B_n(s^n+s^{-n})+\ \ldots\ldots\ldots(38).$$

$$2(1+e^2-es-es^{-1})^{-\frac{5}{2}}=C_0+C_1(s+s^{-1})+C_2(s^2+s^{-2})+C_3(s^3+s^{-3})$$
$$+\ldots\ldots+C_n(s^n+s^{-n})+\ \ldots\ldots\ldots(39).$$

$$2(1+e^2-es-es^{-1})^{-\frac{7}{2}}=D_0+D_1(s+s^{-1})+D_2(s^2+s^{-2})+D_3(s^3+s^{-3})$$
$$+\ldots\ldots+D_n(s^n+s^{-n})+\ \ldots\ldots\ldots(40).$$

Differentiating (37), (38), and (39) we get

$$e(1-s^{-2})(1+e^2-es-es^{-1})^{-\frac{3}{2}}=A_1(1-s^{-2})+2A_2(s-s^{-3})+3A_3(s^2-s^{-4})$$
$$+\ldots\ldots+nA_n(s^{n-1}-s^{-n-1})+\ldots\ldots(41).$$

$$3e(1-s^{-2})(1+e^2-es-es^{-1})^{-\frac{5}{2}}=B_1(1-s^{-2})+2B_2(s-s^{-3})+3B_3(s^2-s^{-4})$$
$$+\ldots\ldots+nB_n(s^{n-1}-s^{-n-1})+\ldots\ldots(42).$$

$$5e(1-s^{-2})(1+e^2-es-es^{-1})^{-\frac{7}{2}}=C_1(1-s^{-2})+2C_2(s-s^{-3})+3C_3(s^2-s^{-4})$$
$$+\ldots\ldots+nC_n(s^{n-1}-s^{-n-1})+\ldots\ldots(43).$$

From (38) and (41) we get

$$\tfrac{1}{2}e(1-s^{-2})[B_0+B_1(s+s^{-1})+B_2(s_2+s^{-2})+\ldots\ldots+B_n(s^n+s^{-n})+\ldots\ldots]$$
$$=A_1(1-s^{-2})+2A_2(s-s^{-3})+3A_3(s^2-s^{-4})+\ldots\ldots+nA_n(s^{n-1}-s^{-n-1})+\ldots\ldots$$

Equating coefficients of s^{n-1} we get

$$2nAn=e(B_{n-1}-B_{n+1})\ldots\ldots\ldots\ldots(44).$$

Writing $(n+1)$ for n we get

$$2(n+1)A_{n+1}=e(B_n-B_{n+2})\ldots\ldots\ldots(45).$$

From (37) and (38) we get

$$(1+e^2-es-es^{-1})[B_0+B_1(s+s^{-1})+B_2(s^2+s^{-2})+\ldots\ldots+B_n(s^n+s^{-n})+\ldots\ldots$$
$$=A_0+A_1(s+s^{-1})+A_2(s^2+s^{-2})+\ldots\ldots+A_n(s^n+s^{-n})+\ldots\ldots$$

Equating coefficients of s^n we get

$$A_n=(1+e^2)B_n-e(B_{n-1}+B_{n+1})\ldots\ldots\ldots\ldots(46).$$

Writing $(n+1)$ for n we get

$$A_{n+1}=(1+e^2)B_{n-1}-e(B_n+B_{n+2})\ldots\ldots\ldots\ldots(47).$$

Eliminating B_{n-1} between (44) and (46) we get

$$(2n+1)A_n=(1+e^2)B_n-2eB_{n+1}\ldots\ldots\ldots\ldots(48).$$

Eliminating B_{n+2} between (45) and (47) we get

$$(2n+1)A_{n+1}=-(1+e^2)B_{n+1}+2eB_n\ldots\ldots\ldots\ldots(49).$$

Eliminating B_{n+1} between (48) and (49) we get

$$B_n=\frac{2n+1}{(1-e^2)^2}[(1+e^2)A_n-2eA_{n+1}]\ldots\ldots\ldots\ldots(50),$$

$$B_{n+1}=\frac{2n+3}{(1-e^2)^2}[(1+e^2)A_{n+1}-2eA_{n+2}]\ldots\ldots\ldots(51).$$

From (39) and (42) we get

$$\tfrac{3}{2}e(1-s^{-2})[C_0+C_1(s+s^{-1})+C_2(s^2+s^{-2})+\ldots\ldots+C_n(s^n+s^{-n})+\ldots\ldots$$
$$=B_2(1-s^{-2})+2B_2(s-s^{-3})+\ldots\ldots+nB_n(s^{n-1}-s^{-n-1})+\ldots\ldots$$

Equating coefficients of s^{n-1} we get

$$2nB_n=3e(C_{n-1}-C_{n+1})\ldots\ldots\ldots\ldots(52).$$

Writing $(n+1)$ for n we get

$$2(n+1)B_{n+1}=3e(C_n-C_{n+2}) \quad \ldots\ldots\ldots\ldots\ldots \quad (53).$$

From (38) and (39) we get

$$(1+e^2-es-es^{-1})[C_0+C_1(s+s^{-1})+C_2(s^2+s^{-2})+\ldots\ldots+C_n(s^n+s^{-n})+\ \ldots\ldots$$

$$=B_0+B_1(s+s^{-1})+B_2(s^2+s^{-2})+\ldots\ldots+B_n(s^n+s^{-n})+\ \ldots\ldots$$

Equating coefficients of s^n we get

$$B_n=(1+e^2)C_n-e(C_{n-1}+C_{n+1})\ldots\ldots\ldots\ldots\ldots(54).$$

Writing $(n+1)$ for n we get

$$B_{n+1}=(1+e^2)C_{n+1}-e(C_n+C_{n+2})\ldots\ldots\ldots\ldots\ldots(55).$$

Eliminating C_{n-1} between (52) and (54) we get

$$(2n+3)B_n=3(1+e^2)C_n-6eC_{n+1}\ \ldots\ldots\ldots\ldots\ldots(56).$$

Eliminating C_{n+2} between (53) and (55) we get

$$(2n-1)B_{n-1}=-3(1+e^2)C_{n+1}+6eC_n\ \ldots\ldots\ldots\ldots(57).$$

Eliminating C_{n+1} between (56) and (57) we get

$$C_n=\frac{1}{3(1-e^2)^2}[(2n+3)(1+e^2)B_n-2e(2n-1)B_{n+1}]\ \ldots\ldots\ldots(58).$$

Similarly,

$$D_n=\frac{1}{5(1-e^2)^2}[(2n+5)(1+e^2)C_n-2e(2n-3)C_{n+1}]\ \ldots\ldots\ldots(59).$$

And generally,

$$Q_n=\frac{1}{(2m+1)(1-e^2)^2}[(2n+2m+1)(1+e^2)P_n-2e(2n-2m+1)P_{n+1}]\ldots\ldots\ldots(60).$$

(50) and (51) in (58) gives us

$$C_n=\frac{2n+3}{3(1-e^2)^4}[(2n+1)(1+e^2)^2A_n-8en(1+e^2)A_{n+1}+4e^2(2n-1)A_{n+2}]\ \ldots(61).$$

$$C_{n+1}=\frac{2n+5}{3(1-e^2)^4}[(2n+3)(1+e^2)^2A_{n+1}-8e(n+1)(1+e^2)A_{n+2}$$

$$+4e^2(2n+1)A_{n+8}]\ldots\ldots\ldots\ldots\ldots(62).$$

(61) and (62) in (59) will give us

$$D_n=\frac{2n+5}{15(1-e^2)^6}[(2n+3)(2n+1)(1+e^2)^3A_n-6e(1+e^2)^2(2n+3)(2n-1)A_{n+1}$$

$$+12e^2(1+e^2)(4n^2-5)A_{n+2}-8e^3(2n+1)(2n-3)A_{n+3}]\ldots\ldots\ldots(63).$$

Knowing the values of A_n, A_{n+1}, etc., we can find the values of B_n, B_{n+1}, etc.; C_n, C_{n+1}, etc.; D_n, D_{n+1}, etc.

[To be Continued.]